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## Heat flux concentration through polymeric thermal lenses

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A significant contributor to energy inefficiency is the generation as well as the uneven dissipation of heat. Practical methods to adeptly channel heat flux ( $Q$ ) would then have widespread applications to improved energy utilization and thermal energy management. It would be beneficial to engineer lens-like composite materials (graded in terms of length or thermal conductivity) with augmented attributes for heat control. Here, we propose and demonstrate polymeric composite based  $Q$  focusing lenses, architected through geometrical considerations. We indicate a five-fold enhancement of the  $Q$ , at the level of  $\sim 2500 \text{ W/m}^2$ , enabled through such thermal lenses. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4904260>]

Much recent work on thermal energy control seeks to understand atomic scale paradigms that underlie transport processes, e.g., phononics, with the broad objective of “manipulating heat flow with electronic analogs and beyond.”<sup>1</sup> However, such methodologies are fraught with fundamental limitations, e.g., the difficulty of fabricating thermal insulators at room temperature, due to the overlap of the low frequency acoustic phonon modes.<sup>2,3</sup> In this letter, an alternate scheme of focusing heat, in analogy to an optical converging lens/light concentrator, is suggested. Motivated by the basic definition of a lens, as one “that reconfigures a transmitted energy distribution,”<sup>4</sup> and its working principles, i.e., through geometric arrangement of a material property such as the refractive index (the analogue of which in the case of heat could be the thermal resistivity or its inverse, the thermal conductivity:  $\kappa$ ), we use correspondent principles. Such thermal energy focusing media can be used in tandem with visible light spectrum harnessing devices, employing wavelength separators, and it will be shown later in the paper how the concentrated heat could be transduced to an electrical voltage through the use of thermoelectric generators (TEGs) (Fig. 1(a)).

Our ideas are nominally distinct from recent theoretical<sup>5</sup> and experimental<sup>6,7</sup> approaches related to the manipulation of  $Q$ , through ideas borrowed from transformation optics.<sup>8,9</sup> While such metamaterials based studies invoke the invariability of fundamental physical constants and conformal mapping<sup>6,10</sup> or coordinate transformation<sup>11</sup> based techniques, we have adapted a variational methodology equivalent to Fermat’s principle,<sup>12</sup> with the hypothesis that the thermal energy propagation would be such that the spatial integral of the thermal resistance ( $R_{th}$ ) over a unit length of propagation ( $dl$ ), i.e.,  $\int R_{th} dl$ , would be a minimum/extremum<sup>13</sup> (where  $R_{th} = 1/\kappa A$ , with  $A$  as the heat flux pertinent area). The minimization would be based on the arrangement of media/materials through which  $Q$  propagation occurs. It is proposed that the constitutive material character (through the  $\kappa$ ) along with extensive parameters such as the thermal traversal length ( $L$ ) and  $A$  can be suitably adjusted to achieve the desired directionality of  $Q$  as well as the extent and magnitude of the focusing. Consequently, thermal energy re-orientation could be accomplished leading to functionalities such as energy focusing or dispersion akin to paradigms familiar from basic optical lens phenomenology.

Designs related to the concentration of heat through variation of the  $\kappa$ , as well as geometry will now be discussed. Experiments coupled with finite element analysis (FEA, using COMSOL<sup>®</sup> Multiphysics) based computer simulations were used to confirm analytical assumptions. We first illustrate a prototype of a thermal concentrator, in analogy with a light-concentrating lens. When exposed to a light beam, the elemental lens unit needs to perform the function of energy focusing, with the spatial extent of the focused spot related to the effective wavelength and the numerical aperture.<sup>14</sup> However, as incoherent heat transfer phenomena is generally<sup>15</sup> wavelength insensitive/diffusive the focal spot would be less well defined. The gradual increase in the optical path length,  $\int n dl$  (where  $n$  is the optical refractive index) from the center to the edge of a typical optical converging lens was mimicked, in our experiments on thermal lenses, through a similar variation of the  $\kappa$ . Ideally, such change needs to be implemented through a very large number of layers and concomitant decrease in the layer thickness ( $t$ )

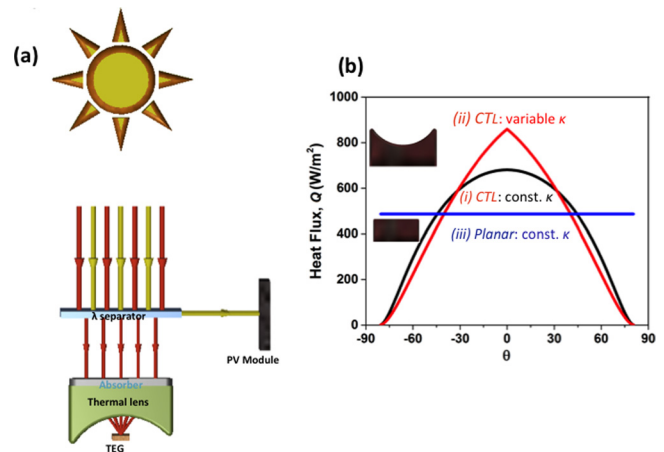


FIG. 1. A concave thermal lens (CTL) (a) constituted from designed polymeric materials, is proposed for efficient harness of low grade heat flux, e.g., the thermal energy from the sun, which may be partitioned through a wavelength segregator and focused onto a thermoelectric generator (TEG). (b) The thermal lens could be fabricated through geometrical arrangement of homogeneous or layered (with linear/non-linear varying thermal conductivity:  $\kappa$ ) material. For a given temperature gradient and geometry, the extent to which heat could be focused was found to be the largest for (ii) a CTL formed using layers with varying  $\kappa$  from the center to the edge, in comparison to a homogeneous  $\kappa$  constituted (i) CTL or (iii) flat/planar geometry.

and would be implicitly related to the assumption of an effective medium for the composite thermal lens. Indeed, it was previously shown<sup>11</sup> that such an assumption is exact for infinitesimally small  $t$ . The thermal lens characteristics could then arise, e.g., (1) due to a change in the  $\kappa$ , keeping the  $Q$  traversal length constant or (2) due to the variation of the geometry in the form of a change of the  $L$ , at a constant  $\kappa$ . While the former case corresponds to a *planar* surface, the latter would imply a *curved/lens-like* geometry.

We first indicate, in Fig. 1(b), the FEA modeled variation of the heat flux for the *lens-like* geometry, where a concave thermal lens (CTL) was composed of either (i) bulk material with isotropic/homogeneous  $\kappa$  or (ii) layers with varying  $\kappa$  from the center to the edge, in comparison to a (iii) flat/planar geometry (with homogeneous  $\kappa$ ). It was clear that, for a given temperature gradient, the  $Q$  enhancement was maximal for (ii). The area under the curves (proportional to the power) in the figure was identical in all the three cases, ensuring the conservation of  $Q$ . The symmetry of the CTL profiles may be related to the dependence of the  $Q$  to  $\kappa(\theta)/L \cos \theta$ , where  $\theta$  refers to the angular position of a particular element of uniform  $\kappa$  along the periphery of the lens, reckoned with respect to the horizontal, at the center of curvature of the CTL. While  $\kappa(\theta)$  was variable for (ii), it was constant for (i).

For experimental verification, polymeric nanocomposites were chosen for the thermal lens materials. The rationale for such a choice was the ability to tune the  $\kappa$  over a narrow range to enable an effective thermal medium (ETM), as well as economic costs coupled with simple fabrication, and could at the very outset be related to the harness of low-grade waste heat. Such composites could relatively easily accommodate a gradual variation (increase) in the  $\kappa$ , which was implemented through the progressive and non-agglomerative<sup>16</sup> addition of carbon nanotubes to a RET (reactive ethylene terpolymer) polymer matrix.<sup>17</sup> The detailed procedures for nanocomposite synthesis<sup>18</sup> and characterization<sup>17</sup> have been previously reported and for our experiments, we used multilayered composites. We compare results of a planar thermal lens (PTL): Fig. 2(a)—simulations and Fig. 2(b)—experiment, with  $\kappa$  decreasing linearly from the center to the edge, to that of a CTL (Fig. 2(c): simulations and Fig. 2(d): experiments) with similar decreasing variation in the layers. 11 layers were symmetrically arranged, i.e., from one edge to the center to the other edge, with individual  $\kappa$  values of 0.34 W/m K, 0.37 W/m K, 0.40 W/m K, 0.44 W/m K, 0.50 W/m K, 0.60 W/m K, 0.50 W/m K, 0.44 W/m K, 0.40 W/m K, 0.37 W/m K, and 0.34 W/m K, respectively. With reference to Fig. 2(c), while a smoothly varying  $\kappa(\theta)$  was desirable, the experimental arrangement yielded more reliable results through the use of wedge-shaped samples. Conformal mapping techniques could be used for mapping the wedges onto circular geometries<sup>19</sup> implying similar geometric characteristics and energy focusing attributes as those of spherical lenses.<sup>20</sup>

For the experimental investigations, the input  $Q$  to the thermal lens was maintained at a uniform and constant value of  $\sim 500$  W/m<sup>2</sup> through a thick block of acrylic. Constant temperatures were maintained at the heat source and sink, employing a hot plate heater and a custom built Peltier cooler (see Sec. I of the supplementary material<sup>29</sup>). A large

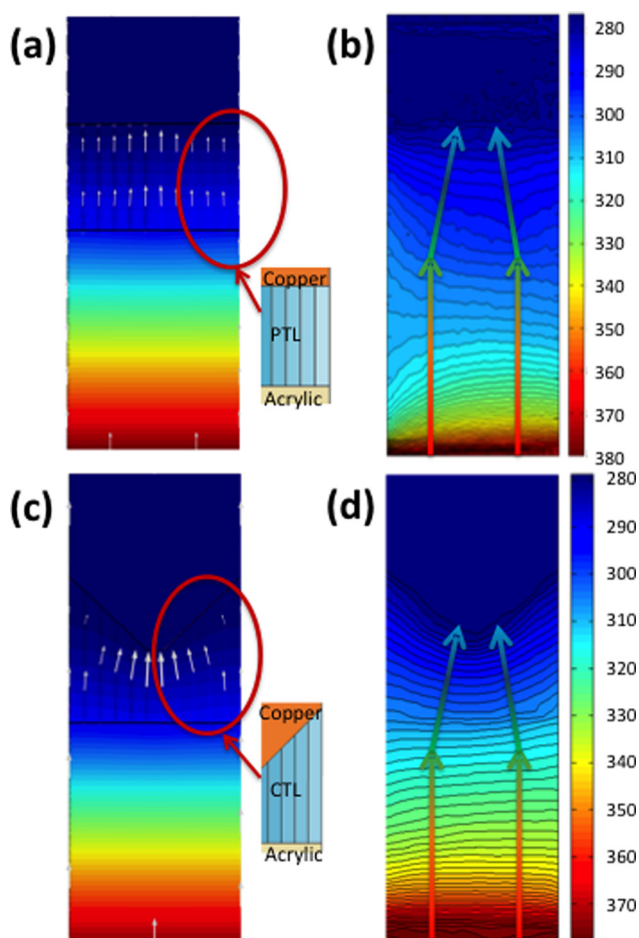


FIG. 2. Heat flux focusing in planar thermal lenses (PTLs): (a) simulations and (b) experiments; and CTLs: (c) simulations and (d) experiments; and thermal lenses. The insets to the figures indicate the respective layered arrangements—typically the  $\kappa$  was varied from 0.3 W/m K in the center to 0.6 W/m K at the edge through the gradual addition of carbon nanotubes to a host polymer. The bottom acrylic (at a temperature,  $T_{hot} = 380$  K) and the top copper (at a lower temperature,  $T_{cold} = 278$  K) block function as high heat capacity thermal energy emitters and collectors. The thermal flux contours (in white) are perpendicular to the isotherms and are increasingly focused towards the center, through the use of CTL geometry.

block length ensured that entrance effects<sup>21</sup> could be ignored. Correspondingly, a copper block with a conforming shape was placed adjacent to the lens surfaces in both the PTL and the CTL arrangements and served as a heat collector. Both the  $Q$  source (acrylic block) and the  $Q$  sink (copper block) function in a manner similar to electrodes for electrical or electrochemical devices, serving as large heat capacity reservoirs. Isothermal conditions were ensured by holding the bottom (the *hot side*) of the acrylic at  $T_{hot} = 380$  K and the top (the *cold side*) of the copper block at  $T_{cold} = 278$  K. The heat conduction temperature profiles were non-invasively imaged, using an infra-red camera (FLIR A15 SC, with a spatial resolution of  $\sim 0.3$  mm and a temperature resolution of  $\sim 0.25$  K), with appropriate calibration (see Sec. I of the supplementary material<sup>29</sup>). To achieve good signal fidelity through enhanced thermal emissivity from the surface of the composite, the top surfaces were painted with a thin layer of black paint of emissivity  $\sim 0.95$ .

The enhanced  $Q$  concentration through the use of the thermal lenses was evident through the FEA simulations (*left*

panels) as well as in the experiments (right panels) of Fig. 2. For the simulations, corresponding to the experimental situation, constant temperature boundary conditions were assumed at the entry and exit, while the sides were insulated. The  $Q$  contours in Figs. 2(b) and 2(d) were oriented to be perpendicular to the isothermal lines—obtained through IR imaging, and clearly indicate a focusing towards the center in both the PTL and the CTL cases, as graphically indicated through the plots of Fig. 3. The extent of  $Q$  concentration was significantly larger through the use of the CTL, which with a five-fold enhancement compared to the input/source flux and a 3.5 times increase (with a center heat flux:  $Q_c \sim 2460 \text{ W/m}^2$ ) compared to the PTL (with  $Q_c \sim 704 \text{ W/m}^2$ ). It was again verified, for both the lenses, that the  $Q$  averaged over the cross-section was  $\sim 502 \text{ W/m}^2$  and  $\sim 505 \text{ W/m}^2$ , respectively—in excellent accord with the flux through the preceding acrylic block (of  $500 \text{ W/m}^2$ ), and as also computed through the ratio of the temperature drop across the acrylic of  $\sim 75 \text{ K}$  (with a  $T_{hot} = 380 \text{ K}$ ) to the thermal resistance:  $L_{ac} (= 3 \text{ cm})/\kappa_{ac} (= 0.2 \text{ W/m K})$ . The heat losses due to convection and radiation were estimated, through simulations, to be  $< 5\%$  of the flux and have been ignored. Through ensuring energy conservation through the composite in the steady state (see Fig. S2 in the supplementary material<sup>29</sup>), such  $Q$  values were intermediate between considering that the individual layers act as parallel or serial thermal resistors, falling within the respective upper and lower Wiener bounds<sup>22</sup> and signifying the ETM character<sup>11</sup> of the thermal lenses. It was evident that the dual use of varying the effective  $Q$  traversal length as well as the  $\kappa$  results in enhanced thermal energy concentration. The discreteness in the  $Q$  profiles of Fig. 3 stems from the finite number of layers used in the composite and can be further smoothed through the use of additional layers corresponding to an ETM. Given the typical correlation of diffusive

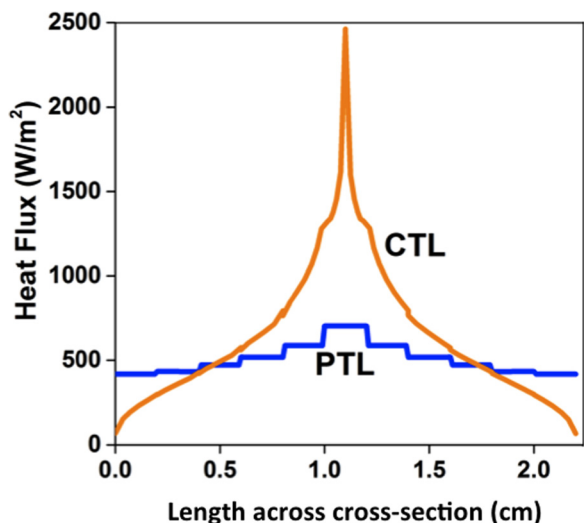


FIG. 3. Enhanced heat flux concentration for the CTL. For an input heat flux of  $\sim 500 \text{ W/m}^2$ , a close to five-fold enhancement in the output flux was observed, at the center of the CTL. A much smaller increase yielding  $\sim 700 \text{ W/m}^2$  was noted for the PTL. The areas under the curves—indicating the net transmitted heat flux, were identical in both the CTL and PTL cases. The discreteness in the profiles was due to a finite number of layers used for the thermal lenses and is expected to transition to a smooth Gaussian profile as the number of layers are increased yielding an effective medium.

processes operating under gradients to Gaussian distributions,<sup>23,24</sup> the thermal energy concentration could be modeled appropriately.

To further correlate the  $Q$  concentration to an increase in the temperature and to examine whether such an increase could be translated to useful energy harness, the arrangements of Fig. 4(a)—for the PTL and Fig. 4(c)—for the CTL were used for conversion into electricity.<sup>25</sup> This was done through the symmetric placement of TEGs along the outer surface of the lens, which were then connected to voltmeters. A calibration of the TEG voltage with temperature (as further discussed in Sec. II of the supplementary material<sup>29</sup>) was used for temperature evaluation at various points along the surface of the respective lenses. A temperature enhancement at the center, of greater than 40% for the PTL configuration (Fig. 4(b)) and greater than 70% for the CTL (Fig. 4(d)), was manifested through TEG based measurements. The observed temperature ratios were well correlated to the ratio of the  $Q$  at the center ( $=Q_c$ ) to that at the edge ( $=Q_e$ ) and may be used as a measure of the focusing ability of a particular lens configuration (see Sec. III of the supplementary material<sup>29</sup>).

Generally, the limit for the focusing spot size in such thermal lenses could be related to a mean phonon wavelength. However, the mean free path is the minimum distance over which a temperature gradient (and a heat flux) could be defined, which could consequently be considered a practical limit. The overall efficiency of focusing, for the given composite, would depend on the convective and radiative heat transfer losses and would be enhanced if such losses

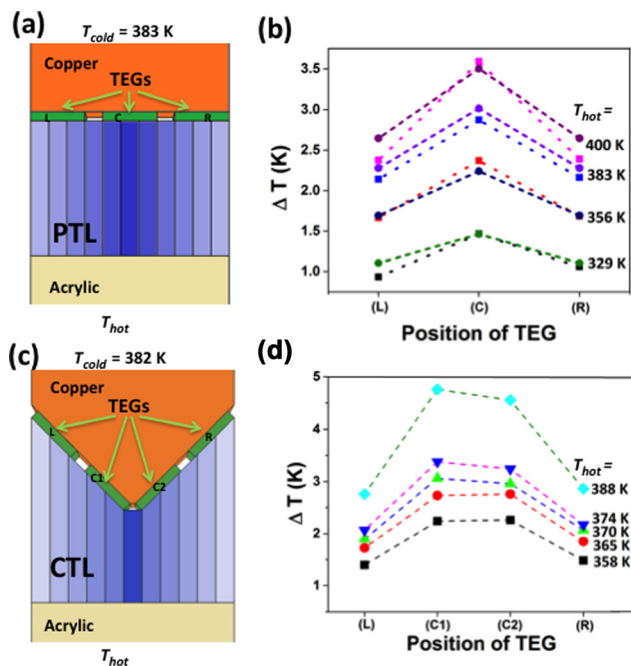


FIG. 4. Heat to electrical energy conversion through thermal lenses. TEGs were placed across the surface of a (a) PTL: top, and a (c) CTL: bottom, constituted from layers with a linearly varying thermal conductivity from the center to the edge. A temperature gradient was applied between the top (with a higher temperature  $T_{hot}$  in the range of 358–388 K) and the bottom (with a constant cold temperature,  $T_{cold} = 282 \text{ K}$ ). The variation of the TEG voltage from the center to the edge, transduced to a temperature difference ( $\Delta T$ ) for (b) a PTL and (d) a CTL, indicates enhanced thermal flux at the center, as designed, and heat to electrical voltage conversion.

could be minimized. The convective loss could be drastically reduced through consideration of the ambient, e.g., if the lenses were placed in a vacuum-like environment. However, radiative losses could be substantial—of up to 20%, for the relatively low heat fluxes considered here. Consequently, adequate radiation shielding should be provided (see Sec. I of the supplementary material<sup>29</sup>). The efficiency may also be improved through alternate materials choice and arrangement, e.g., a large thermal conductivity contrast ratio ( $\kappa_{center} / \kappa_{edge}$ ) would be related to a greater concentration of the heat flux—directed towards the higher thermal conductivity layer and implying greater heat flux concentration efficiency.<sup>26</sup>

Our study has then indicated the development of thermal lenses based on a thermal resistance minimization principle, leading to the concentration of heat flux. The subject area could also be further developed through analogies to concepts extant for diffusive light.<sup>24</sup> Consequently, the presented work constitutes a proof of principle, and many issues related to thermal mode characterization,<sup>15</sup> interfacial thermal resistance,<sup>27</sup> heat flux conversion efficiency, etc., need to be better understood. The layering of elements in a CTL with an exponential variation<sup>13</sup> in the  $\kappa$  could, for example, yield a sharper peak and  $Q$  concentration. Additionally, the use of graded material with larger average values and contrast in the constituent  $\kappa$  would result in a proportionally larger magnitude of the  $Q$  that could be focused and truly<sup>28</sup> help in improving thermal energy utilization.

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<sup>1</sup>N. Li, J. Ren, L. Wang, G. Zhang, P. Hanggi, and B. Li, *Rev. Mod. Phys.* **84**, 1045 (2012).

<sup>2</sup>A. Majumdar, *Microscale Thermophys. Eng.* **2**, 5 (1998).

- <sup>3</sup>D. G. Cahill, W. K. Ford, K. E. Goodson, G. D. Mahan, A. Majumdar, H. J. Maris, R. Merlin, and S. R. Phillpot, *J. Appl. Phys.* **93**, 793 (2003).
- <sup>4</sup>E. Hecht, *Optics* (Pearson/Addison Wesley, San Francisco, CA, 2002).
- <sup>5</sup>S. Guenneau, C. Amra, and D. Veynante, *Opt. Express* **20**, 8207 (2012).
- <sup>6</sup>R. Schittny, M. Kadac, S. Guenneau, and M. Wegener, *Phys. Rev. Lett.* **110**, 195901 (2013).
- <sup>7</sup>S. Narayana and Y. Sato, *Phys. Rev. Lett.* **108**, 214303 (2012).
- <sup>8</sup>J. B. Pendry, D. Schurig, and D. R. Smith, *Science* **312**, 1780 (2006).
- <sup>9</sup>J. B. Pendry, A. Aubry, D. R. Smith, and S. A. Maier, *Science* **337**, 549 (2012).
- <sup>10</sup>S. Guenneau and C. Amra, *Opt. Express* **21**, 6578 (2013).
- <sup>11</sup>K. P. Vemuri and P. R. Bandaru, *Appl. Phys. Lett.* **103**, 133111 (2013).
- <sup>12</sup>U. Leonhardt and T. Philbin, *Geometry and Light: The Science of Invisibility* (Dover Publications, Inc., Mineola, NY, 2010).
- <sup>13</sup>A. Tan and L. R. Holland, *Am. J. Phys.* **58**, 988 (1990).
- <sup>14</sup>B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, 2nd ed. (John Wiley & Sons, Hoboken, NJ, 2007).
- <sup>15</sup>J.-J. Greffet, R. Carminati, K. Joulain, J.-P. Mulet, S. Mainguy, and Y. Chen, *Nature* **416**, 61 (2002).
- <sup>16</sup>S. Pfeifer and P. R. Bandaru, *Mater. Res. Lett.* **2**, 166 (2014).
- <sup>17</sup>R. S. Kapadia, B. M. Louie, and P. R. Bandaru, *J. Heat Transfer* **136**, 011303 (2013).
- <sup>18</sup>S.-H. Park, P. Theilmann, P. Asbeck, and P. R. Bandaru, *IEEE Trans. Nanotechnol.* **9**, 464 (2010).
- <sup>19</sup>A. D. Wunsch, *Complex Variables With Applications* (Pearson/Addison Wesley, Boston, MA, 2005).
- <sup>20</sup>U. Leonhardt, *Science* **312**, 1777 (2006).
- <sup>21</sup>T. L. Bergman, A. S. Lavine, F. P. Incropera, and D. P. Dewitt, *Introduction to Heat Transfer* (John Wiley, Inc., Hoboken, NJ, 2011).
- <sup>22</sup>G. W. Milton, *The Theory of Composites* (Cambridge University Press, Cambridge, UK, 2002).
- <sup>23</sup>D. S. Lemons, *An Introduction to Stochastic Processes in Physics* (The Johns Hopkins University Press, Baltimore, MD, 2002).
- <sup>24</sup>J. Goodman, *Statistical Optics*, 1st ed. (Wiley-Interscience, New York, NY, 2000).
- <sup>25</sup>P. Pichanusakorn and P. R. Bandaru, *Mater. Sci. Eng. R* **67**, 19 (2010).
- <sup>26</sup>T. Yang, K. P. Vemuri, and P. R. Bandaru, *Appl. Phys. Lett.* **105**, 083908 (2014).
- <sup>27</sup>E. T. Swartz and R. O. Pohl, *Rev. Mod. Phys.* **61**, 605 (1989).
- <sup>28</sup>D. J. MacKay, *Sustainable Energy—Without the Hot Air* (UIT Cambridge Ltd., Cambridge, UK, 2009).
- <sup>29</sup>See supplementary material at <http://dx.doi.org/10.1063/1.4904260> for experimental details on thermal setup and FEA simulations and calibration of thermoelectric elements and temperature.